

## Voltage Mode Control: The Modulator in Continuous Current Mode (CCM) of operation

There are three main components in the control loop of a voltage mode DC to DC converter. The three stages are the output stage consisting of the output filter, the modulator gain, and the voltage loop compensation. The two topologies for generating a PWM signal are fixed ramp voltage and feed forward voltage topology both of which will be discussed in this document. The issue of how the modulator gain stage affects the stability of the voltage mode open loop system and closed loop system in continuous current mode (CCM) will also be discussed.

### 1. Basics of operation and definitions

The actual PWM signal in a voltage mode regulator is generated by a comparator triggering on a voltage ramp as shown in diagram 1. This ramp is generated from a clock signal and it can be fixed to a particular peak voltage or it can be variable depending on  $V_{in}$  as in the feed forward topology.

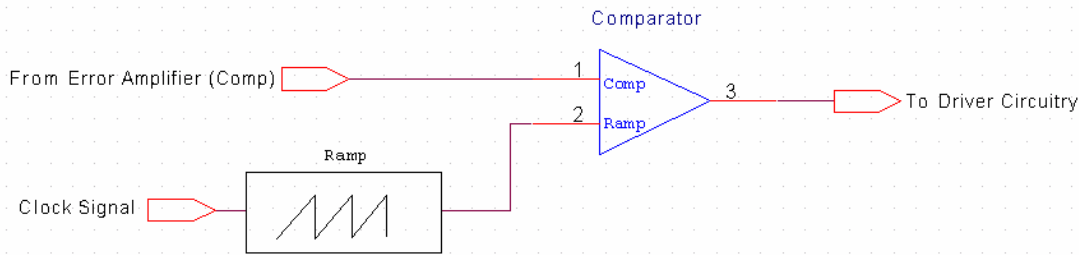


Diagram 1

When the error amplifier input ( $V_{comp}$ ) is 0 V the duty cycle is 0% meaning the part is off. When the error amplifier voltage ( $V_{comp}$ ) is equal to peak of the ramp voltage ( $V_{ramp}$ ) the driver circuitry is at 100% duty cycle.

The duty cycle of the controller is defined as  $D=t_{on}/T$  where T is the Total time defined with respect to the comp voltage and the ramp voltage.

$$D = \frac{t_{on}}{T} \quad (1)$$

$$\begin{aligned} t_{on} \text{ time} &= V_{comp} \\ t_{off} \text{ time} &= V_{ramp} - V_{comp} \\ T &= t_{on} + t_{off} \end{aligned}$$

This results in equation 2 by substituting  $t_{on}$  and T into the duty cycle equation 1

$$D = \frac{V_{comp}}{V_{comp} + V_{ramp} - V_{comp}} = \frac{V_{comp}}{V_{ramp}} \quad (2)$$

It is important to note that these definitions could vary from controller to controller depending on what the driver logic is but for the SP613X family and the PowerBlox family of products the above definitions hold true.

The gain of the modulator is defined as

$$Gain = \frac{\partial V_{out}}{\partial V_{comp}} \quad (3)$$

In the feed forward topology another definition is needed. In a feed forward topology the ramp voltage is no longer fixed but varies depending on what the input voltage is. The  $V_{ramp}$  signal can go full input voltage range or in many instances it is clamped to a maximum voltage. For example the SP612X family of products has a K (gain) of 5 with a maximum ramp voltage of 3V. Thus up to 15V in the IC has feed forward above that voltage it becomes a fixed ramp voltage of 3V.

$$V_{ramp} = \frac{V_{in}}{K} \quad (4)$$

Where K is a constant

The feed forward topology has advantages over a fixed ramp voltage topology which will be further examined after the Modulator Gains are defined. The constant can be any number chosen at the time of the initial IC definition.

## 2. The Buck regulator topology

The buck regulator is one of the most common topologies used. It is well known how the output voltage is related to Input voltage in equation 5.

$$V_{out} = V_{in} \cdot D \quad (5)$$

D is the Duty Cycle

Substituting Equation 2 into Equation 5

$$V_{out} = V_{in} \cdot \frac{V_{comp}}{V_{ramp}} \quad (6)$$

To get the gain we need to take the derivative of equation 6 with respect to  $V_{comp}$ .

$$Gain = \frac{V_{in}}{V_{ramp}} \quad (7)$$

For a modulator with feed forward another step is needed to get the proper gain, equation 4 needs to be substituted into equation 7. The result for feed forward modulator gain of the buck converter is

$$Gain_{feedforward} = \frac{V_{in}}{\frac{1}{K} \cdot V_{in}} = K \quad (8)$$

### 3. Boost Regulator

Another widely used topology is the Boost regulator. This topology will be investigated here in some detail since it is a little more complicated than the buck regulator. Once the boost regulator gain is solved, all of the other topologies can be derived in the same manner. The boost regulator  $V_{in}$  to  $V_{out}$  relationship is as follows:

$$V_{out} = V_{in} \cdot \frac{1}{1-D} \quad (9)$$

Substituting equation 2 in for duty cycle in equation 9 we get

$$V_{out} = V_{in} \cdot \frac{1}{1-D} = V_{in} \cdot \frac{1}{1 - \frac{V_{comp}}{V_{ramp}}} = \frac{V_{in} \cdot V_{ramp}}{V_{ramp} - V_{comp}} \quad (10)$$

The next step is to take the derivative of equation 10 we get the following results:

$$Gain = \frac{\partial V_{out}}{\partial V_{comp}} = \frac{V_{in} \cdot V_{ramp}}{[V_{ramp} - V_{comp}]^2} \quad (11)$$

Substitute equation 2 after solving for  $V_{comp}$  into equation 11

$$Gain = \frac{V_{in} \cdot V_{ramp}}{[V_{ramp} - D \cdot V_{ramp}]^2} = \frac{V_{in}}{V_{ramp} \cdot (1-D)^2} \quad (12)$$

For a feed forward topology substitute equation 4 into 12.

$$Gain_{feedforward} = \frac{K}{[1-D]^2} \quad (13)$$

## 4. The Topologies summarized

Table 1 shows more voltage mode modulator gain values for different topologies. These were derived the same way as the steps outlined previously in this document. The modulator gains are for continuous conduction current mode (CCM).

TABLE 1

Topology	Duty Cycle Relationship	Modulator Gain Fixed Ramp	Modulator Gain Feed forward
Buck	$V_{out} = V_{in} \cdot D$	$\frac{V_{in}}{V_{ramp}}$	K
Boost	$V_{out} = V_{in} \cdot \frac{1}{1-D}$	$\frac{V_{in}}{V_{ramp} \cdot (1-D)^2}$	$\frac{K}{[1-D]^2}$
Negative Buck Boost	$-V_{out} = V_{in} \cdot \frac{D}{1-D}$	$\frac{V_{in}}{V_{ramp}} \cdot \frac{1}{(1-D)^2}$	$\frac{K}{[1-D]^2}$
SEPIC*	$V_{out} = V_{in} \cdot \frac{D}{1-D}$	$\frac{V_{in}}{V_{ramp}} \cdot \frac{1}{(1-D)^2}$	$\frac{K}{[1-D]^2}$
Cuk*	$V_{out} = -V_{in} \cdot \frac{D}{1-D}$	$\frac{V_{in}}{V_{ramp}} \cdot \frac{1}{(1-D)^2}$	$\frac{K}{[1-D]^2}$

## 5. Effects on the compensation for a Buck Regulator

To see what effect the modulator has on the output filter one has to look at the open loop Bode plot for the topology. Diagram 2 is the typical open loop filter and modulator Bode plot for a buck regulator. The modulator gain and the filter gain are added together to get the total open loop gain for the filter. The important thing to note is that in a fixed ramp topology, as  $V_{in}$  increases so does the total gain of the open loop filter. This in turn pushes out the crossover frequency of the filter which in turn affects the positioning of the poles and zeros of the compensation network. This problem does not exist when the gain of the modulator is fixed as it is in the feed forward topology.

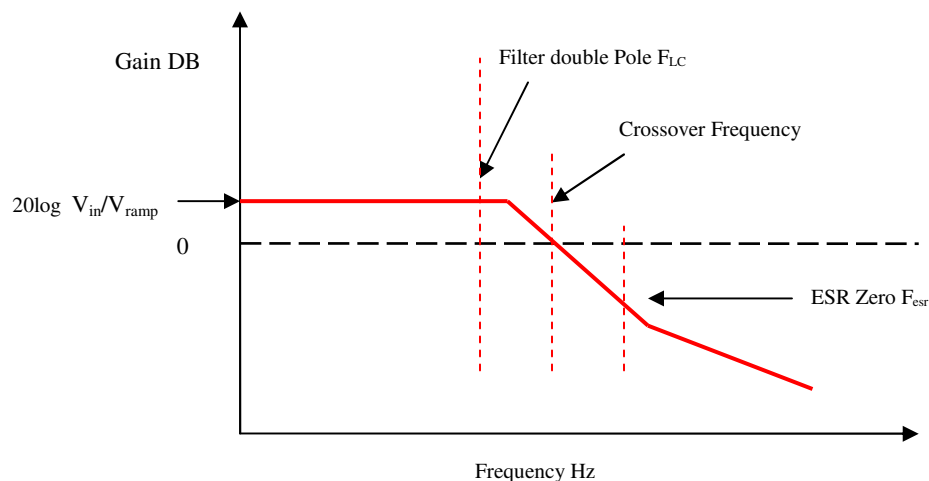


Diagram 2 Open Loop Filter Bode Plot

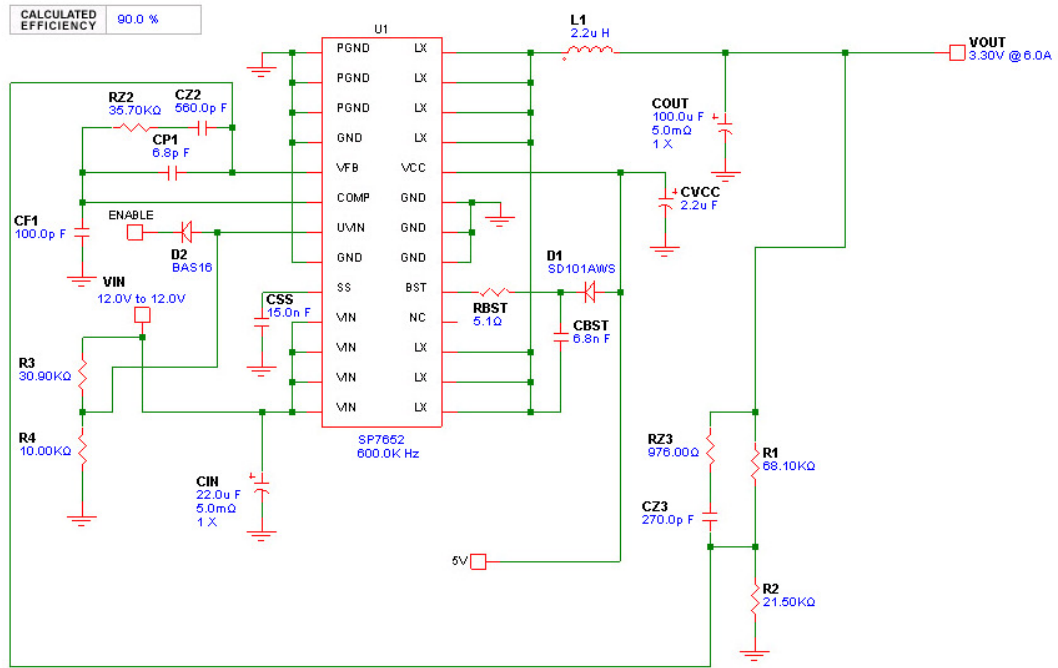


Diagram 3 Schematic for AC Analysis Simulations

In diagrams 4 5 and 6 we have Bode plots for phase and gain for a compensation network for a fixed ramp voltage regulator. The compensation was not altered between different simulations for the different input voltages. The compensation was designed for a 12V input.



Diagram 4:  $V_{in} = 12V$ ,  
Crossover frequency = 73 KHz

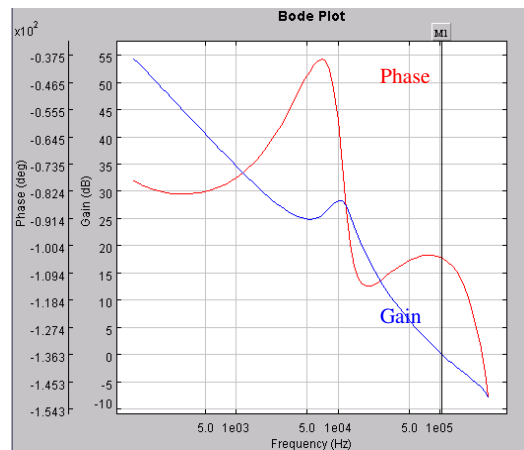


Diagram 5:  $V_{in} = 16V$ ,  
Crossover frequency = 100 KHz

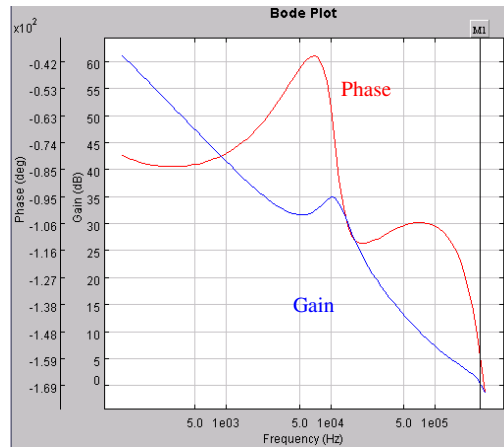


Diagram 6:  $V_{in} = 28V$   
Crossover frequency = 200 KHz

As can be clearly seen from diagrams 4, 5 and 6 as the input voltage increases so does the gain of the modulator. This in turn increases the crossover frequency. There are two well known effects on compensation when the crossover frequency increases but the compensation network does not change.

1. The first is that the crossover frequency should be no more than 1/2 of the switching frequency. This is dictated by the Sampling Theorem to try to avoid aliasing. Typically in a design 1/5 or 1/10 of the switching frequency is used as a crossover frequency. In diagram 4 it can be seen that the crossover frequency is 200KHz at 28V in.
2. The second one is that the phase margin will deteriorate as the crossover frequency increases, which will affect the stability of the converter, as can be seen Diagram 4. This is because the poles and zeros were set for a different crossover frequency to give phase boost in a certain frequency range to have proper phase margin. As the crossover frequency increases the phase will eventually decrease which will make the converter unstable.

This  $V_{in}$  gain issue is negated in the feed forward topology since the Gain is a constant K.

## 6. How to compensate a buck with large input voltage swings without feed forward

In the feed forward topology this is not as much of a problem, but in a fixed ramp topology to get good stable compensation with a large input voltage swing the designer needs to consider how the modulator affects the overall gain of the system. In a buck regulator this means that the compensation design needs to be done at the highest  $V_{in}$  since this will give the designer the highest gain. This gain should be the one used to set the crossover frequency. This approach will guarantee that the regulator will be stable over the whole input range. The drawback of this is that as  $V_{in}$  decreases, so will the crossover frequency, thus degrading the transient response. This will be most noticeable when the input voltage range is rather large.

## 7. Modulator Effects on the open loop Bode plot of a Boost Regulator

The boost converter is a lot more complicated when it comes to compensation and the effects of the modulator on the open loop gain. The problem is that the duty cycle has a big effect on both the filter and the modulator. The effect on the modulator is the same as in the buck regulator Bode plot, the modulator and output filter responses are added together. In Diagram 7a and 7b are graphs showing the modulator Gain with respect to the duty cycle for a boost regulator for specific input voltages.

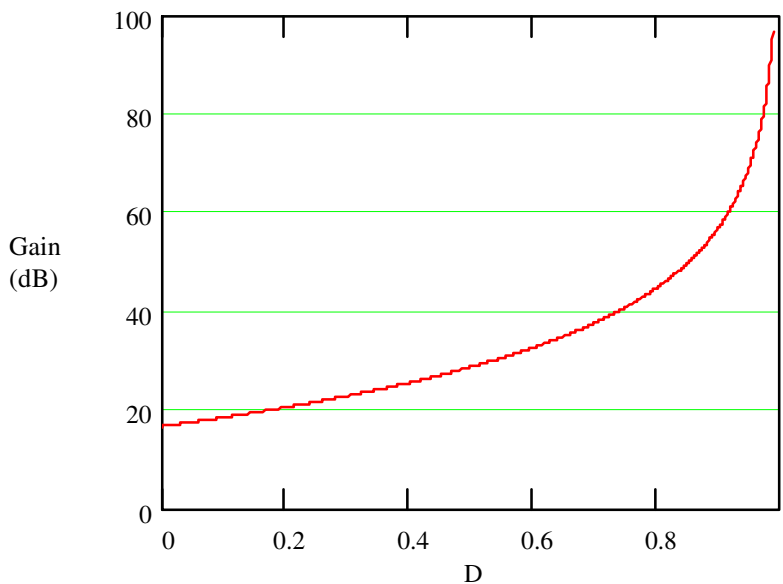


Diagram 7a Modulator gain for  $V_{in}$  of 7V and  $V_{ramp}$  1V (From Table 1)

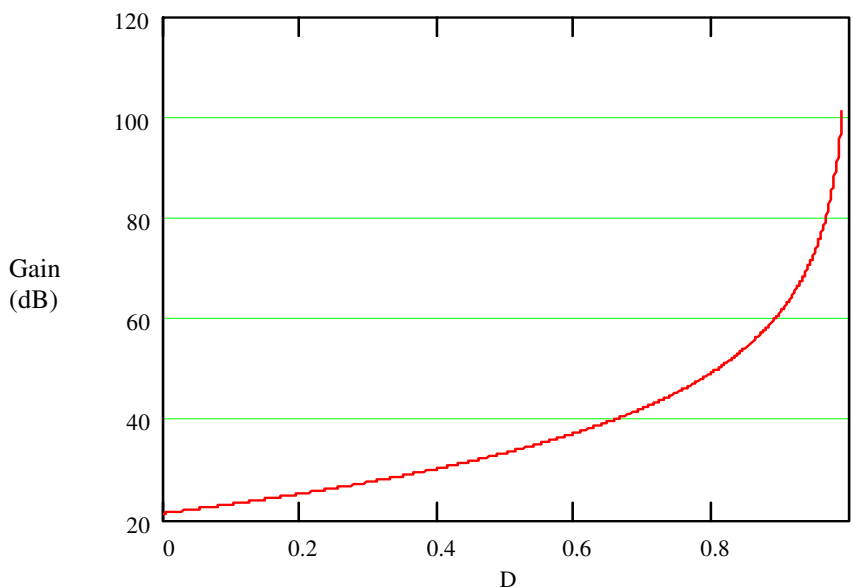


Diagram 7b Modulator gain for  $V_{in}$  of 12V and  $V_{ramp}$  1V (From Table 1)

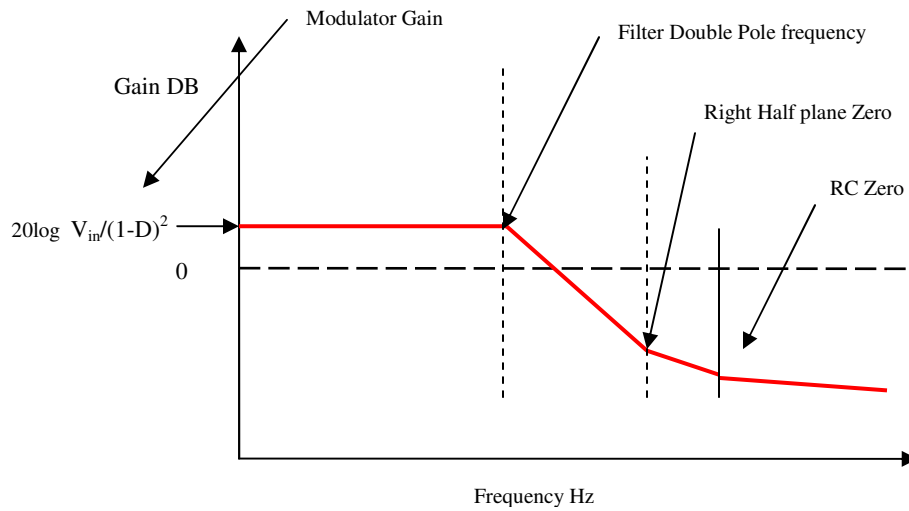


Diagram 8 Open Loop Transfer Function of a boost converter in CCM mode

Filter Double Pole  $\omega_{LC} = \frac{1-D}{\sqrt{L \cdot C_{out}}}$

RC zero  $\omega_{RC} = \frac{1}{ESR \cdot C_{out}}$

Right Half Plane Zero  $\omega_{RHP} = \frac{(1-D)^2 \cdot R_L}{L}$

Modulator Gain  $M_{db} = \frac{V_{in}}{(1-D)^2}$



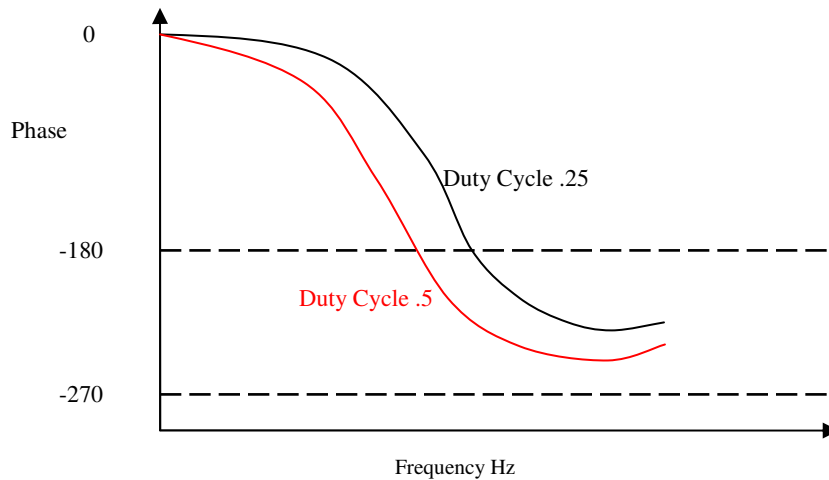


Diagram 9 Phase of a boost regulator

As can be seen from diagrams 7 and 8 and 9, it is not that straightforward when it comes to compensation of a boost. The generic Bode plot of a boost converter in Diagram 8 clearly states that the duty cycle plays an important role in defining the open loop gain.

As to the gain of the modulator it can significantly differ depending on the duty cycle, for  $V_{in}$  of 12V, at 25% duty cycle the gain of the modulator is about 25dB; at 75% percent duty cycle it is about 45dB. Another major thing to consider is that the modulator will have a gain that is approaching infinity as the duty cycle approaches 100%. The other complication is that the gain curve exists for every input voltage of the boost regulator, thus complicating the design even more. Usually as input voltage changes, so will the duty cycle of the design thus creating more variables to deal with. One saving grace is that for a fixed output the duty cycle decreases as the input voltage increases, which tends to help keep the modulator gain in check. This can be seen clearly seen from equation 14 and diagrams 7a and b.

$$D = 1 - \frac{V_{in}}{V_{out}} \quad (14)$$

When a feed forward topology is implemented, the design can be greatly simplified by having one modulator gain curve for a specific voltage range and thus only having to deal with the variations of the duty cycle on the analysis. (Table 1)

## 8. Compensating the Boost Converter

Unlike the buck converter it is much harder to compensate the boost converter for wide input voltage swing when the output voltage is fixed, since changes in  $V_{in}$  will affect the duty cycle of the converter. This type of topology typically is better suited to running in narrower duty cycle ranges with compensation that is not very aggressive. To properly compensate the regulator the

designer needs to make sure that the crossover frequency for his system occurs before the Right Half Plane Zero ( $RHP_{zero}$ ) at a slope of -20dB. It is the right half plane zero that contributes to a sharp decline in phase approaching -270 degrees as seen in diagram 9. The obvious problem there is that the location of the  $RHP_{zero}$  varies with duty cycle and load. Although this can be discussed in more depth, the focus of this paper is on the modulator effects on the Bode plots. Thus unlike the buck compensation, it needs to be considered earlier in the design since the best results are when the duty cycle variation is limited to small changes around the 50% duty cycle point. At this point is where the gain of the modulator is at its most linear with changes in gain of 10 to 20db either way from 50% duty cycle point.

## 9. Summary

The modulator gain has a significant effect on the total open loop gain of the Bode plots. It is also one of the least talked about topics in control loop theory. Hopefully this paper helped the reader understand a small but crucial part of the compensation network.

## 10. Bibliography

- 1) Fred C. Lee, A CPES Professional Short Course at Virginia Tech Lecture Note, © 2005
- 2) Wu. Keng C. Pulse Width Modulated DC-DC Converters. New York: Chapman & Hall, 1997.

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